Platonic Solids and Euler’s Formula

Description:
A hands-on and minds-on activity to discover Euler’s Formula for solids

Objective:
On completion of this lesson, students should gain conceptual understanding of Euler’s formula for polyhedron.

Materials:
Polydron [Polydron, Ltd.] materials are commercial tools that provide students with precise models to build three-dimensional geometric figures. When these materials are not available, teachers can provide alternative models such as cardstock nets and sets of pre-cut congruent regular polygons. Possibly regular polygon cut-outs.

Procedure:
For a detailed description, please see the lesson plan.
Students explore five platonic solids and discuss what they observe about their properties. Students continue their exploration using a tool available on the NCTM website. Next, students construct (or investigate pre-made models) other shapes such as pyramids, prisms and anti-prisms. Within the lesson, students verify Euler’s Formula.

Standards:
National Council of Teachers of Mathematics Standards Grades 6-8

Numbers and Operations:
- Work flexibly with fractions, decimals, and percents to solve problems.
- Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line.

Geometry:
- Describe, classify, and understand relationships among types of two and three dimensional objects.
- Describe sizes, positions, and orientations of shapes under informal transformations (flips, turns, slides, scaling, etc.)

Measurement:
- Select and apply techniques/tools to accurately find length, area, and angle measures precisely.
- Solve problems involving scale factors, using ratios and proportion.
Data Analysis and Probability
- Observe differences between two or more samples to make conjectures about the populations
- Use conjectures to formulate new questions and plan new studies to answer them.

Problem Solving:
- Build new mathematical knowledge through problem solving
- Solve problems that arise in math and other contexts.

Reasoning and Proof:
- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate math conjectures.

Communication:
- Organize and consolidate mathematical thinking through communication
- Analyze and evaluate the mathematical thinking and strategies of others.

Connections:
- Recognize and use connections among math ideas.
- Understand how math ideas interconnect and build on one another to produce a coherent whole.

Representation:
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.

Ohio Academic Content Standards: Mathematics at the Benchmark Level:
Grades 5-7:
D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Grades 8-10:
A. Formally define geometric figures.
E. Draw and construct representations of two- and three-dimensional geometric objects…
Platonic Solids and Euler’s Formula

**Big Idea:** A hands-on and minds-on activity to discover Euler’s Formula for solids

**Specific learning Objectives:** On completion of this lesson, students should gain conceptual understanding of Euler’s formula for polyhedra including the Platonic solids.

**Teacher Background:**
This is a good lesson follow up to The Five Platonic Solids lesson, but can begin immediately following Zome contructions of any convex polyhedra. Teacher should also be familiar with Euler’s formula, which states that for any convex polyhedra the number of faces plus the number of vertices is always equal to the number of edges plus 2.

**Methods**
1. Pair students and distribute models of the five Platonic solids or provide nets, Polydron, or Zometools for construction. With either the pre-made models or the student constructed models, provide time for the students to explore and discuss what they observe about their properties. Students are encouraged to note patterns, observe the individual structures and make generalization about the regularities across the solids.

2. Have the students use the models to guide them in their investigation of the relationships among the faces, vertices and edges. Engage students in exploration and analysis by asking probing questions such as: Record as many similarities and differences between and among the Platonic solids as you can. Guide the students to determine, which ones have triangular faces? Square faces? Pentagonal faces?

3. Guide the students to create an organizational format to keep track of their discoveries about the relationships between and among the five Platonic solids. Suggestions to students [if needed] would be to organize the discoveries on a chart, perhaps suggesting columns for number of faces, vertices and edges of each solid. (Example: for the tetrahedron; faces = 4; vertices = 4 and edges = 6.)

4. Ask students to construct (or think of) other shapes such as pyramids, prisms and anti-prisms. Include those on the chart.

5. Have students explore the chart in pairs and brainstorm a formula that can be made about these figures.

6. Have a class discussion to see if more than one group discovered that F + V – E = 2.

**Management and Safety Issues**
Be sure that enough models are available for each pair of students.
Circulate among the students while they are working to get a sense of their developing conceptual knowledge.

Encourage multiple ways for students to record and share their discoveries. Particular attention should be given to the organizational structure of their information.

**Materials/Technology**

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**Misconceptions**

Students will often make a generalization without thoroughly exploring their ideas. This often leads to faulty conjectures. Calculation errors may occur with poorly constructed models which may lead to imprecise conclusions.

Although other methods of construction of the polyhedric solids exist, the Zome and Polydron sets prove to be the most mathematically accurate.

Basic algebraic skills are necessary to show that a group with the formula \( F + V - E = 2 \) has the same formula as a group with \( F + V = E + 2 \).

**Assessment**

Observe the students and review their charts. Students should be able to communicate their conjectures and be able to provide supporting arguments for what they have discovered and generalized. To meet the standard they must be able to count the components of the polyhedra and platonic solids accurately and construct a general formula for Euler’s formula.

**Applications/Connections**

The aesthetic and historical aspects of mathematics should be highlighted. Plato speculated that these five solids were the shapes of the fundamental components of the physical universe (EnchantedLearning.com).

Knowledge of polyhedral construction support work in drafting, architecture, building, quilting, graphic design and related fields.

**Research Base/References**


EnchantedLearning.com

NCTM *Mission Mathematics II series* Grades 3-5; 6-8; and 9-12.

NCTM *Navigation Series Navigating Through Geometry* Grades 3-5

*Zome Geometry Book*, Key Curriculum Press

**Equity**

Provide ample time for students during the exploration/discovery phase. Be sure all groups get to share their ideas.

Special needs educators and mathematics classroom teachers should collaborate to plan instruction and select appropriate materials to maximize each student’s opportunity to experience success.

All students’ ideas should be accepted and discussed even if they represent faulty reasoning. Teachers should make clear that the ideas are at stake, not the students who suggest them.

**Standards**

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