

How Tall is Building?

National Council of Teachers of Mathematics Standards Grades 6-8

(summarized - a complete list can be found in the lesson plan linked above):

Numbers and Operations:

- Work flexibly with fractions, decimals, and percents to solve problems.
- Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.

Algebra:

- Relate and compare different forms of representations for a relationship.
- Develop an initial conceptual understanding of different uses of variables.

Geometry:

- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
- Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

Measurement:

- Use common benchmarks to select appropriate methods for estimating measurements.
- Understand relationships among units and convert from one unit to another within the same measuring system.

Problem Solving:

- Build new mathematical knowledge through problem solving
- Apply and adapt a variety of appropriate strategies to solve problems.

Reasoning and Proof:

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Develop and evaluate mathematical arguments and proofs.

Communication:

- Organize and consolidate mathematical thinking through communication
- Use the language of mathematics to express mathematical ideas precisely.

Connections:

- Recognize and use connections among math ideas.
- Recognize and apply mathematics in contexts outside mathematics.

Representation:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.

Ohio Department of Education Academic Content Standards: Mathematics Grades 5-7

Number, Number Sense, and Operations Standard

- Use a variety of strategies to estimate, compute, solve, and explain solutions to problems involving integers, fractions, decimals, and percents.

Measurement

- Identify appropriate tools and apply appropriate techniques for measuring.
- Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time, and temperature.

Geometry and Spatial Sense

- Apply properties of equality and proportionality to solve problems involving congruent or similar figures.

Mathematical Processes Standard

- Relate mathematical ideas to one another and to other content areas.
- Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
- Use representations to organize and communicate mathematical thinking and problem solving.

Description

In this lesson, students will look at three different approaches to measuring the same thing by using standard and non-standard measuring devices. Given a specified building, students will use the shadow method, the triangle method, and the thumb method to determine the height of this building. Discussion will follow as to the advantages and disadvantages of each method.

Objectives

1. Students will estimate the height of a given building.
2. The class will brainstorm ways as to how to determine the height of the given building and form some conjectures.
3. Students will demonstrate and understanding of three strategies for measurement: similar right triangles, a right isosceles triangle used to sight and object, and repeated addition.
4. The students will analyze the results of all three methods in order to determine the height of the building and the precision of measurement and discuss the advantages/disadvantages of each.

Materials

1. Student worksheets
2. Newprint papers, paste, tape, and markers

- 3. 3x5 inch index cards
- 4. measuring tapes
- 5. trundle wheels
- 6. calculators
- 7. computer with spreadsheet program
- 8. assessment sheets

Procedure

This is a brief summary of the activities for this lesson - for a more detailed description please refer to the full lesson plan available at the link above

- 1. Estimation writing - Students will think about the problem, work in groups to share and communicate their thoughts and ideas
- 2. Instructor presentation of the three strategies to be used in this lesson: similar right triangles, a right isosceles triangle used to sight on an object, and repeated addition.
- 3. Students work in newly assigned groups to determine the height of the building using the three different strategies.

Lesson Plan

Big Idea of this Lesson:

Students will look at three different approaches to measuring the same thing by using standard and non-standard measuring devices. Given a specified building, students will use the shadow method, the triangle method, and the thumb method to determine the height of this building. Discussion will follow as to the advantages and disadvantages of each method.

Specific Learning Objective: Grades 6 – 9 Time Frame: 200 minutes

Objectives

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- 2. The class will brainstorm ways as to how to determine the height of the given building and form some conjectures.
- 3. Students will demonstrate and understanding of three strategies for measurement: similar right triangles, a right isosceles triangle used to sight and object, and repeated addition.
- 4. The students will analyze the results of all three methods in order to determine the height of the building and the precision of measurement and discuss the advantages/disadvantages of each.

NCTM Standards for Grades 6 - 8:

1. Numbers and Operations:

- Work flexibly with fractions, decimals, and percents to solve problems.
- Understand and use ratios and proportions to represent quantitative relationships.
- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods.
- Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results.
- Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

2. Algebra

- Relate and compare different forms of representations for a relationship.
- Develop an initial conceptual understanding of different uses of variables.
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations.

3. Geometry

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
- Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
- Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.
- Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.
- Draw geometric objects with specified properties, such as side lengths or angle measures.
- Use geometric models to represent and explain numerical and algebraic relationships.
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

4. Measurement

- Understand both the metric and customary systems of measurement.
- Understand relationships among units and convert from one unit to another within the same measuring system.
- Use common benchmarks to select appropriate methods for estimating measurements.
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision.

- Solve problems involving scale factors, using ratio and proportion.

5. Problem Solving:

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

6. Reasoning and Proof:

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

7. Communication:

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

8. Connections:

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in context outside of mathematics.

9. Representation:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

Ohio Department of Education Academic Content Standards: Mathematics Grades 5-7

Number, Number Sense, and Operations Standard

- Use a variety of strategies to estimate, compute, solve, and explain solutions to problems involving integers, fractions, decimals, and percents.

Measurement

- Identify appropriate tools and apply appropriate techniques for measuring.
- Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time, and temperature.

Geometry and Spatial Sense

- Apply properties of equality and proportionality to solve problems involving congruent or similar figures.

Mathematical Processes Standard

- Relate mathematical ideas to one another and to other content areas.
- Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
- Use representations to organize and communicate mathematical thinking and problem solving.

Methods: This learning activity can be divided into three parts with assessments at the conclusion of each part.

1. Estimation writing: Think, Pair, Share, and Communicate.

- Begin by asking each individual student to estimate the height of the given building. Have them include a drawing of the building and a scale for that drawing. Ask each student to communicate the process used to determine the answers on their his/her individual paper.
- Ask students to share their findings with their partners to compare the estimations and scales.
- Move students into a new assigned group of four. In this group, develop a group estimation process paper that shows the building, a scale, and the estimated height. Attach each of the four individual papers to the group paper and hang these up around the room.
- Using the walk around and question method, give each group an opportunity to view the work of the other groups and write questions that need clarification on the group paper. Then have each group present their estimation work to the class and answer the clarifying questions.
- Assessment: Each student writes a justification essay about the process and gives an estimation for the height of the building with supporting evidence for the decision.

2. Teacher presentation of the three strategies to be used in this project: similar right triangles, a right isosceles triangle used to sight an object, and repeated addition.

- Using the lesson sheets entitled, *Three Strategies for Measurement of Height*, which follow, present each strategy.
- Have each group demonstrate one of the three strategies to the class. Let the desired height to be found be the height of the classroom ceiling. The shadow method may be dropped if not appropriate to the conditions of the room.
- Formulate a conjecture, use a model, draw a graphic, and communicate the mathematics used to determine the height of the ceiling.
- Each group will present their findings to the class and a chart will be made of the results. Precision needs to be connected to the answer in the discussion of the height

of the ceiling. Listening to each presentation is demonstrated.

- Look at the data and come to a consensus as to the height of the room and the precision of accuracy.
- Using a tape measure, determine the height of the ceiling and note its precision.
- Discuss the advantages and disadvantages of each strategy.
- Assessment: Have each group answer these questions. How can these strategies be extended to be used to determine the height of the given building? What would the process look like for each of the three strategies? What might be the pitfalls of each strategy? How can these pitfalls be avoided?

3. Now find it!! Students work in new assigned groups to determine the height of the building using three different strategies.

- Each group will formulate an estimation of the height of the building and the precision of accuracy for that measure.
- Using the worksheets entitled, *Just how tall is that building?*, the new assigned groups of students will determine the height and precision of the accuracy of each measurement. Drawings will be made. Communication of the mathematics used will be illustrated.
- A group newsprint poster will be produced showing the estimation, each of the strategy papers with the drawings and results. The group's final decision as to the height of the building with the precision of accuracy will be stated. These will be posted around the room.
- Using the walk around and question method, give each group an opportunity to view the work of the other groups and write questions that need clarification on the group paper. Then have each group present its work to the class and answer the clarifying questions.
- Record on a spreadsheet the results of each strategy from each group and determine the overall average. Discuss the validity of this as the answer for the actual height of the building.
- Discuss the advantages and disadvantages of each strategy.
- Assessment: Each student will write two reflective essays. The first one is to describe the group process used to determine the final answer as to the height of the building. The second one is to justify the value of this experience by answering the question, identify the value that this experiment had for you.

Management Issues:

1. Use a building with a large flat area around it that can be used to do the activity.
2. The activity needs to be done on some days with enough sun to cast a shadow.
3. Following directions is essential.
4. Be certain that each student is familiar with the operation of a trundle wheel.
5. Students must also be familiar with a tape measure. Encourage students to use the metric measurements but do not limit it as their only choice.
6. Students need to be familiar with conversion between units of measure in the same

measuring system prior to the activity.

Materials/Technology:

1. Class Lesson Sheets and Student Worksheets named : *Just how tall is that building?*
2. Newsprint papers, paste, tape, and markers
3. 3 by 5 inch index cards
4. measuring tapes
5. trundle wheels
6. calculators
7. computer for spreadsheet access
8. assessment sheets: Justification Assessment, Extension Assessment, Reflective Assessment found in the Student Worksheets

Misconceptions:

1. Conversion of measurement with the trundle wheel that has a circumference of one meter and measures off in clicks.
2. The process of converting all measurements into the same unit of measure.
3. What to do with the constant distance from the ground to the measurer's eye level.
4. The process of using the thumb for repeated addition as a nonstandard measuring device.

Assessment: Each of the three sections has two types.

1. Class discussion based upon final report that includes advantages and disadvantages of each method.
 2. Individual or group written assessment as stated above in the methods section.
- (Assessment sheets have been provided in the Student Worksheets.)

Applications:

1. To determine the height of a building, a tree, or wire from the ground, these strategies can be used.
2. Use in the surveying of land.
3. In astronomy, the use of the thumb strategy as apparent distance to actual distance is common for measuring the distance to the moon or a star.

Research Base/References:

Cleveland Collaborative for Mathematics Education (C2ME), Problem-Solving Infusion Project: Problem Solving Units for middle Grade Students. (1994) The Cleveland Education Fund, Cleveland, OH.

Equity:

- Teacher assigns groups.
- Math literacy is essential. (ie. Hypotenuse, leg, trundle, base, metric system)
- The different measuring tools for each strategy that will be used by each student
- A teacher aide may be necessary for a student with special needs.