The discovery of topological defects and topological states of electronic matter are among the great discoveries in the condensed matter physics of the past fifty years. I will review these developments and describe how analogous effects may be found in the elastic properties of two-dimensional materials. Experiments on bilayer graphene sheets have observed fault lines that are the mechanical analog of domain walls in magnets and point-like defects that are the mechanical analog of vortices in superconductors. We have developed a continuum elasticity theory that describes these topological defects. Our theory also predicts that bilayer graphene is the mechanical analog of a topological insulator with gapless edge modes at boundaries that are analogous to the electronic edge states of topological insulators.