Queuing Theory and the Taguchi Loss Function: The Cost of Customer Dissatisfaction in Waiting Lines

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Abstract

As customer's wait longer in line they become more dissatisfied. Because a wait time of zero is not economical, a balance must be obtained between the cost of waiting and the cost of service. Classic queuing theory does not generally provide cost figures. Therefore, in order to quantify the cost of waiting, this article utilizes the Taguchi loss function, which was developed for physical products, to determine the cost of customer dissatisfaction. Specifically, this article combines the M/M/1 queuing model with the Taguchi loss function to establish the cost of customer dissatisfaction.

Introduction

The cost of quality, as related to both products and services, is something that is often difficult to quantitatively measure. For example, the amount of time a customer is required to wait in line prior to service is a key component of the level of quality perceived, and therefore a determinant of customer satisfaction. Obviously, some costs are incurred when a customer becomes dissatisfied. However, because these costs are not readily quantifiable and sometimes remain unknown, many organizations do not perform a cost/benefit analysis considering a dissatisfied customer and the resources needed to provide more or better services. Using the Taguchi Loss Function, the cost of a dissatisfied customer can be approximated to provide management with information regarding when changes should be made to either improve waiting times or at least lower the perception of the wait. The following provides an overview of the theory behind the Taguchi Function and its integration into waiting lines analysis. A detailed example follows that explains how such a cost/benefit analysis can be used to determine the cost of customer dissatisfaction.

Taguchi Loss Function and Waiting Lines

The Taguchi Loss Function was derived by Genichi Taguchi in the late 1950s in Japan. Previous quality models had argued that no cost to the organization or the consumer was incurred unless the product went beyond its upper or lower specification limits (USL or LSL, please see Figure 1). Once the upper specification limit is reached it is assumed that

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costs incurred will remain constant regardless of how large the deviation is from the specification limit, or the target value.

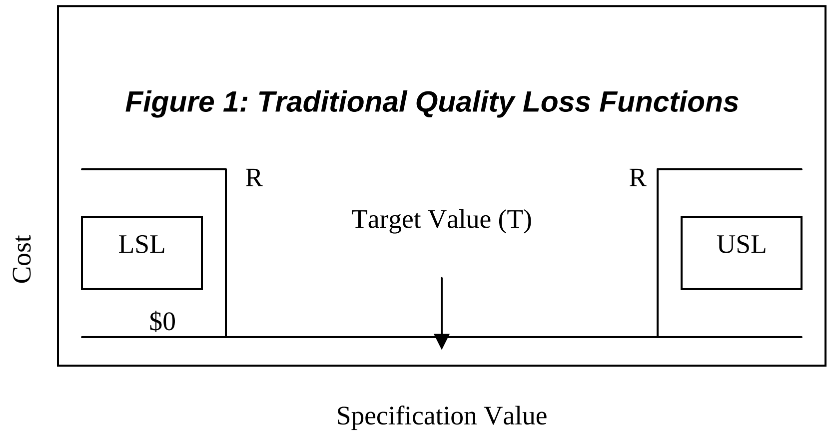

Figure 1: Traditional Quality Loss Functions

In Figure 1, R is the cost of rejection at the specification limit. The traditional cost function (C) is

$$C = \begin{Bmatrix} 0 \; if \;\; LSL \leq m \leq USL \\ R \;\; otherwise \end{Bmatrix},$$

where m is the measurement of the quality characteristic.

The Taguchi Loss Function takes a different perspective on when the costs of poor quality are incurred. Taguchi theorized that rather than incur costs beginning at two finite points that are +/- a specific level of tolerance from the target value (or specification nominal value), costs are actually incurred as soon as the value moves from its target value. In addition, rather than continue at a constant rate, these costs are incurred at the square of the deviation from the target value, and therefore continue to increase the farther the specification deviates from the targeted value. The only point in the model at which no loss is incurred is at the actual targeted value. In contrast with traditional models, the Taguchi Loss Function is represented in Figure 2.

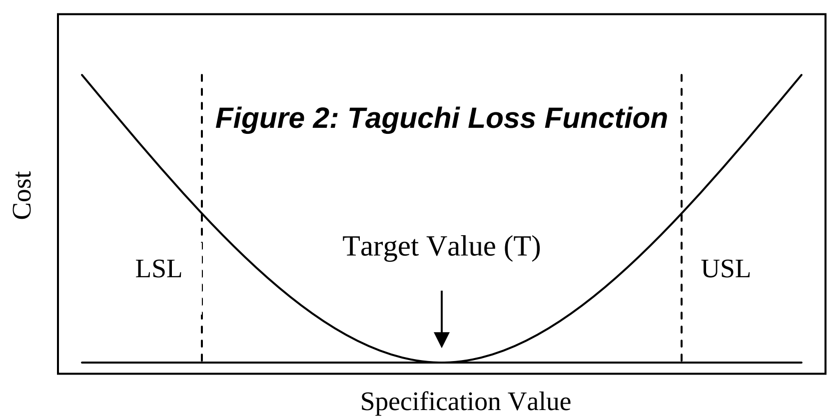

Figure 2: Taguchi Loss Function

Therefore, the cost of quality (C) is

$$C = K(m - T)^2,$$

where $K = \dfrac{R}{(USL - T)^2}$,

and T is the target (or specification nominal) value. R is the cost of rejecting the item at the specification limit. Specifically K is determined from the cost of rejecting the item at the specification limit, and the distance from the target value to the specification limit.

As one can see, the Taguchi model presents an improved opportunity for estimating the costs incurred as a result of poor product or service quality. In previous analyses the model has been used to evaluate the costs and benefits of quality enhancing capital investments (Fink, Margavio, and Margavio, 1994), and selecting suppliers (Quigley and McNamara, 1992).

The model discussed herein shall combine the Taguchi loss function with the components of waiting lines and customer dissatisfaction. Waiting line or queuing theory is utilized primarily to assess the trade-offs between the cost of improving service and the cost of customer waiting time, which is directly linked to customer satisfaction. Two of the traditional performance measures that waiting line models determine include waiting time (the amount of time the customer spends waiting prior to receiving service), and time in system (the combined amount of waiting and service time). The time spent in line or in system is tied to customer satisfaction—short wait times result in satisfied customers, while long wait times translate to dissatisfied customers. Unfortunately, a perfect system in which there is no wait time is not economically feasible for organizations. Therefore, organizations must trade-off the cost of service versus the cost of customer dissatisfaction from waiting.

Traditional queuing models are not effective at measuring the cost of waiting. However, by using the Taguchi Loss Function to determine the costs of customer dissatisfaction due to waiting in line and comparing these costs to those associated with adding or improving service indicated as necessary through queuing models, one can better understand the cost/benefit relationship between the cost of service and the cost of waiting. The following model and example illustrate this theory.

Model Derivation

The model described herein utilizes the equations used to calculated time in line and time in system for a single channel waiting line (M/M/1). The M/M/1 queuing model assumes that there is one line with one server (e.g., one cashier), and that the arrival rate distribution is Poisson and the service time distribution is exponential. Further, customers are selected from the line in a first come, first served fashion.

By combining the Taguchi Loss Function with the appropriate queuing equation provides the user with a method for calculating the cost of customer dissatisfaction associated solely with the time spent waiting for service. Note that only the positive side of the Taguchi loss function is used, since waiting time is only one-sided (a negative wait time is impossible). Two derivations are provided, one using time in line, and the other using time in system. In some cases, the customer is only concerned with the time in line. For example, at an amusement park, the time in line is the primary concern. Most customers would prefer that the ride last longer, which would make the time in system longer. In other situations, the customer's concern is getting through the system as fast as possible. When you car is in the shop, you are primarily concerned with getting it back. Therefore, time in system would be the preferred measure.

Derivation of Time in Line

The pdf (probability density function) of time in line (Gross and Harris, 1974) is:

$$f(t) = (1 - \rho) + \int_0^\infty \lambda(1 - \rho)e^{-\mu(1-\rho)t}dt,$$

note that (1-ρ) is the probability of no waiting time. It can be dropped from the analysis since no (zero) waiting time will not have a cost associated with it. Also note that ρ is the utilization, and is ρ=λ/μ, where λ is the arrival rate and μ is the service rate. For the pdf to hold, μ (service rate) must be greater than λ (arrival rate). Also, t represents the time in line. Using this pdf with the Taguchi loss function, the expected cost per customer using the Taguchi function for time in line (C_q) is

$$C_q = \int_0^{\infty} \lambda(1-\rho)e^{-\mu(1-\rho)t} Kt^2 dt\ .$$

The following illustrates the derivation to a closed form solution:
Let: $U = -\mu(1-\rho)t$.

$$t = \frac{U}{-\mu(1-\rho)}$$

$$dU = -\mu(1-\rho)dt$$

$$dt = \frac{dU}{-\mu(1-\rho)}$$

Substituting the above equations into expected cost per customer using the Taguchi function for time in line produces:

$$C_q = \int_0^{\infty} \frac{-\lambda}{\mu} \frac{U^2}{[-\mu(1-\rho)]^2} e^U KdU$$

$$= \int_0^{\infty} \frac{-K\rho}{[-\mu(1-\rho)]^2} U^2 e^U dU$$

$$= \frac{-K\rho}{[-\mu(1-\rho)]^2} \left[U^2 - 2U + 2\right] e^U \Big|_0^{\infty}$$

$$= \frac{-K\rho}{[-\mu(1-\rho)]^2} \left[(-\mu(1-\rho)t)^2 - 2(-\mu(1-\rho)t) + 2\right] e^{-\mu(1-\rho)t} \Big|_0^{\infty}$$

$$= \frac{2K\rho}{[-\mu(1-\rho)]^2}$$

$$= \frac{2K\rho}{[\lambda-\mu]^2}$$

Derivation of Time in System:

The *pdf* (probability density function) of time in system (Gross and Harris, 1974) is:

$$f(t) = \mu(1-\rho)e^{-\mu(1-\rho)t}$$

The expected cost per customer using the Taguchi function for time in system (C_s) is:

$$C_s = \int_0^{\infty} \mu(1-\rho)e^{-\mu(1-\rho)t} Kt^2 dt$$

Let: $U = -\mu(1-\rho)t$.

$$t = \frac{U}{-\mu(1-\rho)}$$

$$dU = -\mu(1-\rho)dt$$

$$dt = \frac{dU}{-\mu(1-\rho)}$$

Substituting the above equations into expected cost per customer using the Taguchi function for time in system produces:

$$C_s = \int_0^\infty -\frac{U^2}{[-\mu(1-\rho)]^2} e^U K dU$$

$$= \int_0^\infty \frac{-K}{[-\mu(1-\rho)]^2} U e^U dU$$

$$= \left[\frac{-K}{[\mu(1-\rho)]^2}\right]\left[U^2 e^U \Big|_0^\infty - 2\int_0^\infty U e^U dU\right]$$

$$= \left[\frac{-K}{[\mu(1-\rho)]^2}\right]\left[U^2 e^U \Big|_0^\infty - 2e^u (U-1)\Big|_0^\infty\right]$$

$$= \left[\frac{-K}{[\mu(1-\rho)]^2}\right]\left[\left(U^2 - 2U + 2\right)e^U \Big|_0^\infty\right]$$

$$= \left[\frac{-K}{[\mu(1-\rho)]^2}\right]\left[\left[(-\mu(1-\rho)t)^2 - 2[-\mu(1-\rho)t] + 2\right]e^{-\mu(1-\rho)t} \Big|_0^\infty\right]$$

$$= \frac{2K}{[\mu(1-\rho)]^2}$$

$$= \frac{2K}{[\mu-\lambda]^2}$$

Costs of Customer Dissatisfaction

To determine the cost of customer dissatisfaction in waiting lines, it is important to know its components. Specifically, the costs related to customer dissatisfaction derived from waiting are both quantitative and qualitative in nature, and include the following:

Lost Sale - Obviously, the retail outlet could lose a potential sale if the customer chooses to leave prior to receiving service. In addition, future sales could be lost if the dissatisfaction level rises so high that the customer decides not to return to the store. Research has also demonstrated that the average dissatisfied customer tells nine to ten people about their poor experience, with thirteen percent telling more than 20 people (Hoffman*)*. If the initial dissatisfied customer bears any influence on the other people informed of the experience, a store could experience further lost sales.

Increased Competition - The faults of one firm in terms of customer satisfaction only serves to further open the door to increased competition. Eventually this leads to further lost sales and market share, as well as increased expenses incurred through marketing campaigns aimed at regaining those customers who have been lost.

Loss of Reputation - Through poor service, as evidenced by increased waiting times, the firm also incurs a costs associated to the decline of its reputation. While the costs incurred in building the reputation are essentially sunk, any associated loss would be considered a devaluation of the firm's worth.

Decline in Employee/Management Performance and Morale – Few want to work for a company with a poor reputation for service. Increased waiting and service times will be viewed by employees as management's perceived inability to provide adequate resources for providing service, including equipment, number of terminals, or additional employees for peak times. Management's position on the cause of customer dissatisfaction likely will focus on the perceived lack of employee productivity. Either position or combination thereof could be correct. Regardless, the result will include increased hiring and training costs through either termination or re-training. Costs may also be incurred through the hiring of additional employees. However, whereas this may be the retail stores short-term solution, through the use of queuing theory models and the Taguchi Loss Function, management could determine the optimal combination of resources necessary to achieve both customer satisfaction and profitability.

Example

The following model utilizes a cashier at a small store to illustrate the concepts presented in the above model. Assume the following:

Arrival Rate	12 customer per hour
Service Rate	16 customer per hour

In addition, assume that if a customer waits 20 minutes that his/her cost of dissatisfaction is $40.00. The $40.00 is costs associated with this instance of customer dissatisfaction. First, calculate the constant K. It should be noted that the 20 minutes of wait time is converted into hours by dividing by 60 minutes in one hour in the equation...

$$
\begin{aligned}
K &= R/(USL-T)^2 \\
&= 40/(20/60-0)^2 \\
&= 360
\end{aligned}
$$

Using the equation derived earlier results in the following Taguchi cost,

Average Time in Line (minutes)	11.25
Taguchi Cost per person (C_q)	$ 33.75
Taguchi Cost per hour	$ 405.00

As can be seen, the Taguchi cost is $405 per hour. This is found by taking C_q by the arrival rate per hour. Normal analysis would only look at traditional queuing performance measures such as time in line. In this case, the 11.25 minutes in line appear to be longer then desired. However, it is not clear how much this time in line is costing the organization. The Taguchi cost indicates that $405 per hour in potential savings exists. If new cash register/check-out equipment can be purchased that increase the service rate from 16 customers per hour to 20 customers per hour, then the following levels of performance would be expected:

Average Time in Line (minutes)	4.50
Taguchi Cost per person	$ 6.75
Taguchi Cost per hour	$ 81.00

The resulting cost savings is $405 per hour from the old process minus $81 per hour from the new process which results in a $324 per hour savings. At this point a cost/benefit analysis could be performed to determine if the investment in new equipment is worth-while.

Reducing the Costs of Customer Dissatisfaction

In this example, the costs of customer dissatisfaction are to be reduced via a reduction of waiting times. There are several additional methods available for achieving this cost reduction.

Increase Service Points – An obvious solution for reducing dissatisfaction costs is to increase the available points of service – more registers, more checkers. However, it would be ineffective for a grocer to increase overall staffing. Rather peak times (early evenings, weekends) should be analyzed to determine if the costs of adding more service points are comparable to the savings achieved via reduced customer dissatisfaction. One current trend within retailing is the use of self-scanning terminals. This allows one checker to manage upwards of four check-out lines. Overall costs to the stores are reduced and customer dissatisfaction decreases through reduced wait times.

Waiting Time Guarantee – Some retailers maintain special offers for those customers who wait more than a given time in line. For example, if the customer waits more than 8 minutes he or she may receive 10% off their order for that day. This method benefits both the store and the customer. While the grocer will reduce its profits via the discount, it will not incur the costs associated with hiring additional staff. Use of coupons/discounts should also increase the customer's tolerance for waiting, as they via the extra wait time as being profitable to them.

Posted Waiting Times – A portion of the irritation associated with waiting in line is due to uncertainty. Estimated wait times can be posted for the customers in order to eliminate some of this uncertainty. When the wait times are known, the customer dissatisfaction will be reduced as their expectations for waiting have been altered. The store runs a risk in this scenario also however, in that if posted wait times are inaccurate, customer dissatisfaction is likely to increase past its original point. Also, in the past customers may not have been more than intuitively aware of the time they are waiting. By providing them with a number, their opinion of the store may change if they see that wait times are consistently higher than they expected. To counter this effect, store employees should be given a waiting time target and should take measures to ensure that the posted time does not exceed a given value. If in fact it does, extra employees should be called to registers in the short term, with management taking other corrective actions (adding more lanes, hiring more employees) to relieve the effects in the long run.

Special Handling of Customer Issues – Often the cause of excessive waiting time in the grocery store check-out line is a customer problem – check processing, alcohol purchase approval, and incorrectly scanning item. Therefore, one of the keys to reducing customer dissatisfaction is to more efficiently handle customer issues without backing up the check-out line. As an example, assume a purchased item cannot be scanned correctly and store personnel must search for the correct price. Rather than force that customer and the rest of the line to simply wait, the checker could use a dual conveyor system to begin the next order while customer service personnel handle the other customer's issue and finish the check-out process. This speeds up the time in the system while attending to the needs of all customers.

"Express" Lane Categories – The concept of the express lane could also be expanded to remedy customer dissatisfaction. Grocery store orders can be classified as small, medium, and large. The traditional express lane is designed to cater to the "small" orders. The customers with medium to large orders are then positioned to fight for the shortest line. Adding check-out lines for those customer with medium size orders (15-30 items) would alleviate waiting times, as well as the irritation of waiting behind someone with an exceptionally large order.

Environment – Environmental factors can also play a role in easing the dissatisfaction associated with waiting in line. These factors include the music played in the store, the color, lighting, or temperature, and physical distractions. Physical factors such as the color, lighting and temperature can be altered to provide a more relaxing environment. This will ease the stress of waiting in line. Also, distractions such as attractive displays, with informational aspects as opposed to just advertising, or televisions featuring in-store advertisements or even the local news can be used to make waiting times seem less.

Conclusion

As one can see from the above analysis, the cost of a dissatisfied customer is not negligible. Waiting in line is a primary source of dissatisfaction. By utilizing well known queuing theories and integrating the theory behind the Taguchi Loss Function, a manager can derive the costs associated with this dissatisfaction. It should again be noted however, that customer dissatisfaction is not just an issue at the upper specification limit, but rather for each moment in time beyond the targeted wait time. Using the Taguchi Function, it can then be seen that these costs increase beyond the upper specification limit. However, by assessing these costs and then taking measures to reduce either the actual or perceived waiting times, organizations can quantitatively determine the cost-benefit relationship of improved waiting lines.

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