Senior Project
Department of Economics

“How Real Interest Rates Affect Housing in California”

Seojin Hylton
May, 2017

Advisor: Dr. Francesco Renna
Abstract

This paper considers how real interest rates affect housing prices in California. After the housing market crisis, the house price index for California dropped to 59% from its peak. In 2017, the index has bounced back to 90% of the peak. Real interest rates are known to be a strong factor of housing prices. In this paper, expected appreciation is considered to be the difference between nominal interest rates and real interest rates. Several models for housing prices will be developed and interpreted; both the types of models and their interpretations will form the present investigation into post-bubble housing prices in California.
**Table of Contents**

I: Introduction ......................................................................................................................... 4

II: Literature Review .............................................................................................................. 6

III: Theoretical Model ............................................................................................................. 10

IV: Data .................................................................................................................................. 11

V: Empirical Model ............................................................................................................... 12

VI: Results .......................................................................................................................... 16

VII: Conclusion ..................................................................................................................... 19

VIII: References .................................................................................................................. 20

IX: Appendix ........................................................................................................................ 22
I: Introduction

The All-Transaction House Price Index for California peaked in the third quarter of 2006, and bottomed-out in the first quarter of 2012 to 59% of the maximum. It has since grown to 90% of the peak (Figure 1). In Figure 1, the United States and California curves are normalized to the median price of houses in the first quarter of 1980 in the United States and California, respectively. The housing market is often on the forefront of the minds of economists and policy makers alike. Indeed, in the United States, housing accounts for 15-18% of the GDP (NAHB), and in California, finance, real estate, renting, leasing, and insurance contribute 21% of the state’s GDP (Statista). Housing prices are closely watched as they often offer a window into health of the economy, and in fact the bursting of the bubble is considered a primary cause of the 2007 recession (Holt, 2009); the cause of the housing bubble was explored in his paper, “A Summary of the Primary Causes of the Housing Bubble and the Resulting Credit Crisis: A Non-Technical Paper.” Before the paper truly begins, the author claims that the bubble’s collapse was a primary cause of the recession of 2007. The trend of mortgage rates since the 1980’s is shown to decrease, and is listed as a partial reason as to how the bubble grew. Among the other reasons are the lower short-term loan interest rates, as well as the lowered standards of the lenders. Perhaps most relevant is his final cause: *irrational exuberance*, which is defined as “a heightened state of speculative fervor.” Despite warnings from government economists, bank economists openly stated that there should be no period of housing price decline, as none had existed in the prior fifty years. Essentially, the expected appreciation could only be positive, which meant houses were always going to be a safe investment. This understanding proved to be false.

Given the event of the bubble, we are curious about reexamining the relationship between housing prices and real interest rates while focusing on the housing market of California.
The housing market in California is peculiar. Throughout recent history, California has had higher-than-average housing price growth. This might not be a big problem if the market wages were similarly above the average in the country. However, it is not, and actually the gap between median home value and median income is widening over time since 2012. In 2014, the median house price in the United States was $286,625, and the median household income was $53,657. The ratio of house price to income is then 5.34. For California, the median house price was $448,750.83 whereas the median income was $60,487. The ratio for California is then 7.41. Therefore, the gap between house prices in California is significantly higher than that of the United States. This makes California a difficult place for new people to move to, and it makes it difficult for single families to finance their own home. In Figure 2, we see a graph with both the

![All-Transaction House Price Index (Index: 1980Q1 = 100)](image)

*Figure 1: All-Transaction House Prices (insert source: FRED CASTHPI/USSTHPI)*
ratio of median house prices to median household income for both California and the United States.

![House Price to Income Comparison](image)

**Figure 2: Ratio of House Price to Median Income**

We see that the housing bubble hit California particularly hard.

**II: Literature Review**

In Harris (1989), the author considers how real interest rates affect housing prices across the entire United States from 1970 to 1985. One would argue that housing prices would decrease due as mortgage rates increase, which would lower the demand. However, Harris points out that, during the 1970's, home prices increased as interest rates (which were steady since the 1950’s) nearly doubled over the decade. This is unexpected behavior for complementary goods. The point of his paper is to unravel why the housing market was behaving this way. It was shown that an increase in nominal interest rate would in fact negatively affect the housing price, and an increase in the real interest rate would increase the housing price; real interest rates raise expectations about future prices and appreciation has a positive impact on housing prices. The effect of the real interest was found to outweigh the effect of the nominal mortgage rates, and
thus Harris was able to explain the first-order observation. He argues that the reason why there was a positive relationship between housing prices and real interest rate is that real interest rates affect expectations. He therefore models housing price as a function of permanent income, occupied housing stock, unoccupied housing stock, nominal mortgage rates, dummy variables for quarterly seasonality, and finally appreciation. Because there is no standard way to measure expectations, Harris estimated four different models for appreciation. Part of the interpretation of Harris’ expectations model involved comparing four approaches to modeling expectations, and then verifying which conformed to economic theory.

1. By assuming that the housing market bases its expectations on inflation, an ARMA model of quarterly changes in consumer price index was calculated. This is an adaptive expectations model.

2. By assuming that appreciation is based directly on past appreciation, expected appreciation can then be modeled by an ARMA model of change in house prices. This is an adaptive expectations model.

3. A distributed lag model of inflation based on past price increases was made. This is his distributed lag adaptive expectations model.

4. The final model, his rational expectations model, uses the distributed lag method to model long-term expected inflation using long-term and short term data. Long term inflation was modeled by 10-year-yield treasury bills, and both CPI and 90-day treasury bills were used for short-term inflation.

Harris found that the distributed lag models followed economic theory, and that the ARMA models did not (the ARMA models suggested that housing is an inferior good). In particular, his adaptive expectations model was the best fit, and he proceeded to analyze the problem with this...
model. His result indicates that there is a positive relationship between appreciation (real interest rates) and housing prices as well as a negative relationship between nominal interest rates and housing prices. The effect of the expectations on housing prices noticeably outweighed the effect of the nominal interest, which explains why at first glance house prices increased with mortgage rates. Among his variables, he found that appreciation was the strongest factor of his overall model of housing prices.

In the paper, “Can interest rates really control house prices?” by Shi, the reasons behind the rapid increase in housing prices in New Zealand between 1999 and 2009 is explored. They use Campbell and Shiller’s linear present value model on house prices and the Gordon’s growth model to investigate the relationship between housing prices and interest rates. They also use a more complicated ARFIMA-type modeling to detect fractional integration (used to allow non-integer lags), and describe a method to detect housing bubbles. They find that there is evidence for less-severe bubbles over their time range, but to check the robustness in their test, they should have included more years’ worth of data. The bubble detection scheme might be very useful in understanding the data regarding California. In line with Harris, their results showed a positive relationship between real interest rates and price growth. Specifically, a 1% increase in interest rates induces a 1.72% increase in housing costs. Shi mentions that ten years of data is not enough. Moreover, he mentions issues with causality: he noted that the relationship between the Official Cash Rate and mortgage rates worked in both directions, and was likely due to bank competition. He also mentions that he was unsure if higher house prices caused increases in lending, or increased lending lead to higher house prices. Towards solving problems, Shi notes that while the Royal Bank of New Zealand did eventually get involved in New Zealand’s housing crisis, they had sufficient information to get involved earlier, and likely prevent economic damage. The
housing price models in Shi were very relevant, but several variables necessary for their model were unobtainable. The rational expectation model introduced is still useful, however, and this will be discussed further.

In Cho’s paper “Interest Rate, Inflation, and Housing Price: With an Emphasis on Chonsei Price in Korea,” the discussion starts with the global economy after the IT bubble, characterized by declining interest rates and increasing housing prices. While this was true in the United States, and particularly in California, it was true in Korea as well: from 2001 to 2003, the housing price index rose by over 30%, and the building construction investment grew by 13.3%. However, the GDP only grew at an average of 4.6%, which is not commensurate with the increase in the housing price. The global housing trends were evident in Korea, yet Korea’s unique chonsei system must be taken into account when analyzing Korea’s housing market as its mechanisms differed noticeably from traditional housing. In Park, chonseis are described as “a rental agreement where the tenant pays a lump sum deposit to the landlord in lieu of rent for two years. The entire deposit (excluding any interest earned) is returned when the household moves out at the end of the tenancy. The amount of the chonsei deposit ranges from 30% to 70% of the housing unit’s market price, depending on market conditions.”

Chonsei holders are not free to move readily, but are somewhat protected from price fluctuations. Hence they must be considered differently from traditional tenants or homeowners. In particular, chonsei holders are likely to not incur losses, but homeowners will make money if housing prices rise, as their house is an investment. In this way, there are options for people to decide how much risk they are willing to incur. A model is introduced to show how chonsei prices are determined, and the model is based on chonsei prices, inflation, and interest rates. However, the model is based on the assumption that there is no housing bubble; indeed, Cho
recognizes the bubble, and explicitly mentions its absence. In Cho, it is shown that chonsei prices depend on the ratio of inflation to interest rates, and in fact the chonsei price rises if real interest declines; this is intuitive. In Shi’s paper, there are factors that are specific to New Zealand, and indeed chonseis are unique to Korea. Therefore, the models developed will not directly apply, but are still useful.

In Guler’s paper, “Housing Prices and Interest Rates: A Theoretical Analysis,” the relationship between housing prices and interest rates takes returns several key results. Among these are that housing stock distribution is very important, but more strikingly, that housing prices do not always have a negative relationship with interest rates. In a particular analysis, Guler shows that vacancy and housing stock rates must be considered along with the moving probability. For example, when the moving probability is 1, then the actual supply is the total number of houses, as everyone is going to move. In simulations, it is found that about 93% of the time, housing prices and interest rates are negatively related. They are positively related 5% of the time. The remaining 2% show times when house prices change due to the supply changing, rather than interest rates. The moving probability is shown to greatly influence this relationship between housing prices and interest rates; indeed, as the probability goes to zero, the percentage of time that the relationship is positive increases from 0% to 7%.

III: Theoretical Model

We expect income, nominal interest rates, and expectations of appreciation to directly influence housing price, which is an indicator of housing demand (Harris). We expect the primary force behind expectations of appreciation to be real interest rates because Harris was able to use real interest rates to describe the housing market, which he originally found to be illogical – specifically, he found out that housing prices decrease with increases in mortgage
rates, but this was overpowered by increases caused by increases in real interest. This is illustrated in Figure 3.

**Figure 3: Theoretical Model Outline**

**IV: Data**

The data used for the paper will be described below, and in a summary table (Table 1). There is overlap between the data used for the two approaches, which will be described below.

Harris used a two-step process for each of his four models: he estimated appreciation, and then used the estimate along with income (INC), house occupancy rates, vacancy rates (VAC), and nominal interest rates (NOM) to model house prices (HP). His data was quarterly, and so he also included dummy variables to account for seasonality. For this paper, the data regard California. There are two key differences; the first of which is that the data for this study is available yearly. The second is that the occupied housing stock per household and vacant housing stock per household were shown to be linearly dependent by SAS, and hence we could
not use both at the same time in the model (they each have the same information). Modeling appreciation uses several variables. These include the yields of ten-year treasury bills (TB10) and 90-day treasury bills (TB3) and the consumer price index (CPI). TB10 is used to estimate long-term inflation, TB3 is used to estimate short-term inflation, and CPI is also used to measure inflation.

The Shi model uses house prices, rental rates, mortgage rates, unemployment rates (E), and the Consumer Confidence Index (CCI). Shi also used the ratio of floating loans to the overall value of all mortgage loans, which was not obtainable; fortunately, his analysis suggests this variable is not significant. He also used a variable called “house lending,” which does not appear to be relevant to mortgages in the United States. This was not defined or expanded on further in his paper, and it was shown to be relevant. This variable is also omitted. The real rents were used ambiguously, that is, it was unclear if they were house rental rates or apartment rental rates. The estimated coefficients were both positive and had p-values less than 0.01. This suggests that they were house rental rates, as apartments are complementary goods. However, house rental rates were not available, so apartment rental rates were used. Many of the data used in Shi’s model are adjusted for inflation, and this was done using CPI and adjusting to 2015 dollars.

When all of the data series are combined and only the overlap is taken, the data range is from 1990 to 2015.

V: Empirical Model
The four expected appreciation models that Harris used were estimated to the best degree possible using available data. All of the following models are shown implemented in SAS in the appendix. After EX1, EX2, EX3, and EX4 are computed, they are used separately to model house prices four times, where housing prices is fitted as:
\[ HP = f(INC, VAC, \text{lag}(VAC), NOM, EX#), \]  

(1)

where \# takes the value 1, 2, 3, or 4. Note that we use one lag of the vacancy data. The data in Equation 1, except for the expectation models, has descriptive statistics in Table 2. This includes the minimum and maximum values, the mean, and the standard deviation. It also includes the year-to-year change, and shows the minimum, maximum, and mean change.

The first model for expected appreciation is a one-step ahead forecasts of CPI based on quarterly changes in CPI. A second-order autoregressive ARMA model with one difference was used. The resulting series is called EX1:

\[
d CPI = \Delta CPI
\]

\[
d CPI_t = \mu + \varphi_1 d CPI_{t-1} + \varphi_2 d CPI_{t-2}
\]

\[
d CPI = 1.94582 + 0.43036 d CPI_{t-1} - 0.17853 d CPI_{t-2}
\]

The second model for expected appreciation assumes that expectations of appreciation are based directly on past appreciation. A fourth degree autoregressive ARMA model on change in house prices was computed, and the resulting series is called EX2:

\[
d HP = \frac{\Delta HousePrice_t}{HousePrice_t}
\]

\[
d HP_t = \mu + \varphi_1 d HP_{t-1} + \varphi_2 d HP_{t-2} + \varphi_3 d HP_{t-3} + \varphi_4 d HP_{t-4}
\]

\[
d HP_t = 1060.8 - 0.01105 d HP_{t-1} - 0.39809 d HP_{t-2} - 0.11132 d HP_{t-3} - 0.36464 d HP_{t-4}.
\]

The third model for expected appreciation is a simple distributed lag model that uses change in housing change to model house appreciation. The series resulting from this adaptive expectations model is called EX3:
\[ Appreciation = f(\Delta \text{HousePrice}) \]  
\[ Appreciation_t = 1.8977 + 0.000231. \]  

The fourth model for expected appreciation is actually about forecasting inflation using long-term indicators (TB10) and short-term indicators (TB3 and change in CPI). It also uses an Almon lag on change in CPI (this is solved for using SAS); this is referred to as \textit{SumINF}. The resulting series of this rational expectation measure is called EX4:

\[ TB10_t = \mu + \varphi_1 TB3_t + \varphi_2 dCPI_t + \varphi_3 \text{SumINF}_t \]  
\[ TB10_t = 337.4864 + 0.410773 TB3_t + 0.025562 dCPI_t - 0.0431 \text{SumINF}_t \]  

In order to understand the results of Equations 2, 3, 4, and 5, an OLS regression of Equation with each is performed. The resulting parameter estimates are shown in Table 3. Note that Table 3 also includes the Shi model, EX5, which will be introduced further in this section.

The Shi paper offers a method for detecting a bubble. It is well known that the housing bubble happened, however this test was performed as a sanity check. An ARIMA(1,0,1) model of house prices is used for a one-step ahead forecast, and the residuals are checked for kurtosis. In Shi, a leptokurtic distribution is given as evidence for a bubble. We used:

\[ HP_t = \mu + \alpha_{t-1} HP_{t-1} + \theta_{t-1} \epsilon_{t-1} \]  
\[ HP_t = 191466.4 + 0.96020 HP_{t-1} - 0.82563 \epsilon_{t-1} \]  

The resulting series from Equation 6 was subtracted from the actual house prices (giving us the residuals). The kurtosis is 11.572, meaning that the data are highly leptokurtic.

Shi’s distributed lags model is:
$$\Delta H P_t = \gamma + \alpha_{t-1} \Delta H P_{t-1} + \sum_{k=0}^{1} \beta_k \Delta d_{t-k} + \sum_{k=0}^{1} \lambda_k \Delta m_{t-k} + \sum_{k=0}^{1} \psi_k X_{t-k},$$

Where $d$ is log real rental rates, $m$ is log real mortgage rates, and $X$ is “other economic variables,” including: $E$, percent change in unemployment rates, and $CCI$, the consumer confidence index. The sums are taken from 0 to 1, to indicate the lags. The current change in log real house prices depends on the lag of change of real log house prices. The real mortgage rates were computed by taking nominal rates and subtracting appreciation. The original model also included the aforementioned variables that were not available. The final form was:

$$\Delta H P_t = 0.000259 + 0.905\Delta H P_{t-1} - 0.22 \Delta d_t + 0.296 \Delta d_{t-1} - 0.0036 \Delta m_t - 0.0038 \Delta m_{t-1} - 0.00151 E_t - 0.00475 E_{t-1} - 0.00399 \Delta I t - 0.00225 \Delta I_{t-1}.$$ (7)

The model in Equation 7 resulted in an $R^2$ value of 0.97, which causes suspicion of a unit-root. We note that the data is already differenced (change in housing price) – it is the difference of the log of real house prices. An Augmented Dickey-Fuller test was used. The p-values for the zero-mean and unit-mean type were 0.01 and 0.05, respectively, so we reject the null-hypothesis that there is a unit root. The p-value for the trend type was 0.22, however the data does not follow a linear trend. For reference, the data is shown below in Figure 4; there is a structure break after 2006 that would prevent linear trending.

The Shi model is another form of expected appreciation. Most of the variables are based on transformations of the original data series. For example, the house prices in Shi’s model are really the difference of the log of real house prices. We will use this model in Harris’ final house price model (Equation 1), but first we will take inverse transforms so that the data is the change in real house price. The transformations and inverse transformations were conducted in Excel, and hence are not reflected in the SAS code in the appendix. The resulting model is labeled EX5.
VI: Results

Via experimentation, the best fit for EX1 was with CPI with a first-degree autoregressive model, which is the same model that Harris made. The model was, statistically, very significant. For EX2, Harris used an ARIMA(4,1,0) model; we used a ARIMA(4,0,0) model as with one difference it did not converge. The results were poor, but tested better than other parameter choices. The estimate for EX3 performed very well, and its p-value was less than 0.0001. In EX4, many of the parameters were shown to be biased, however its $R^2$ value was 0.91.

Next, we consider the results of the housing price equation (see Table 2). These are the outcomes of modeling house prices with the alternative formulation of expectations. For the estimation using EX1, EX2 and EX3, we noticed a few unexpected results. For example, In EX1, we see that house prices ought to decrease as expected appreciation increases, and this result is significant at the 1% confidence interval. This makes it easy to discard EX1. Harris also had counterintuitive signs on his model of EX1, but not on the expected appreciation. According to the model estimated using EX2 and EX3, house prices increase with mortgage rates, which is both traditionally and usually unreasonable. We also notice that appreciation is not a particularly strong contributor. EX3 was the best performing model for Harris, and leads to counterintuitive results in the post-bubble Californian housing market. It is possible that the indicators for
appreciation (EX2 and EX3) did not show strong enough results, and thus the mortgage rate variable compensated; this is the behavior that caused Harris to write his paper in the first place. We discard EX1, and turn our attention to the rational expectations model.

EX4 is the only estimation that gives us results with the potential to make ordinary economic sense. We see that house prices do increase with income, but not drastically. Given that house prices have increased greatly while median income has not, this results is in line with stylized facts. The current vacancy stock does not strongly affect the housing prices, but its lag value certainly does. Housing prices do indeed decrease with mortgage rates, and finally appreciation is a strong positive contributor. We note that the minimum, average, and maximum of EX4 are 1.65, 6.2, and 13.2, respectively. Appreciation is a big positive contributor to housing, as expected, and if the expected appreciation of the house increases by one percent, then the house price increases by $112,044. This raises a red flag, as a 1% increase in expected appreciation likely cannot account for a 23.5% increase in the house price (at 2015 values). At the maximum value of EX4, 13.2%, the model suggests the house price would increase by $6,269,754. At the minimum value of 1.65%, the increase is still by $785,918. The other issue is that EX4 is generally trending downwards, and does not show any noticeable peaks during the bubble. We note that the while considered a percentage, the range is from 0-100, not 0-1.

Vacancy is a huge driver of house prices, but the effect is lagged. Probably this means that it takes time for the market to adjust to a percent change in vacancy. Recalling Guler’s paper, and the importance of the moving probability, we claim this likely is because of the high demand of housing in California. A 1% increase in last year’s vacancy rates will cause a tremendous drop of $471,248, which is almost the entire median house price. It is therefore important to consider the variance of vacancy, which is 0.438 – thus the standard deviation is 0.6625. The maximum year-
to-year difference is 0.2607, and the average change is 0.01. Therefore, the model predicts vacancy causes an average change of $4,866, however the biggest change due to vacancy was a change of $122,862. This occurred between the years 2013 and 2014, and between 2013 and 2015, house prices increase by $68,133. House prices grew by $94,620 between 2013 and 2016. Therefore there is some truth to this number.

A 1% increase in mortgage rates causes a large decrease in prices, probably because they are already high. A 1% increase in mortgage will decrease the price by $77,673. The maximum change was 2.76%, and the average change was -0.16. This lends credibility to the coefficient.

Turning our attention to the Shi model, we see several similarities and differences to Shi’s result. For California, the change in house prices increases when last year’s change in house prices increases. This is the opposite result of Shi, however is in agreement with the irrational exuberance mentioned in Holt. We recall the Shi’s model did build in the bubble. House prices did decrease as mortgage rates increased, which is promising. Housing prices decreased as unemployment increased, which is to be expected. Interestingly, the house prices decreased as consumer confidence increased. The strongest effects, by far, were from the previous year’s change and the change in rental prices (both current and lagged). In Shi, the rental rates had positive coefficients, leading us to believe that he used house rental rates. However, we only had apartment rates available (a complementary good), and as the current year’s rentals increased, housing prices decreased, however an increase in last year’s rental rates leads to an increase in this year’s housing price change. This is likely a comment on how quickly the market reacts to changes.

We now turn our attention to modeling house prices (Equation 1) using the results of Shi's model (Equation 7). The coefficient for median household was similar - $19.67 for EX4, and
$14.44 for Shi. The coefficients for vacancy and the lag of vacancy for Shi are smaller than for Harris' EX4, but they are similar in size. As noted, the largest change in vacancy was by -0.26, and this causes house prices to increase by $77376.77. The big difference is, as it was for EX2 and EX3, the sign for mortgage rates is positive. We note that there is similarity between Shi's model and EX3: EX3 is only based on change in house price, and Shi's is based on last year's change in house price and other factors. In Shi, the expected appreciation does reflect the bubble more than EX4.

VII: Conclusion

In Harris, he found that adaptive expectations was the best model for house prices, and that the rational expectations was a close second. However, Harris’ paper was published before the housing bubble. The Shi model uses past appreciation as well as other common economic indicators when predicting appreciation. The only viable Harris model, using EX4, is based off of several inflation metrics, rather than something directly related to housing. By reusing his methods we find that while the rational expectations model appears to provide the best fit of the Harris candidates, it might not properly capture appreciation.

The Shi paper provides a method to use the available to detect the known bubble, which succeeds. Only the models that are based on inflation (EX1 and EX4) indicate that housing prices decrease when nominal mortgage rates increase. This implies that when an expectation model does falls short of actual expectation, the OLS regression compensates using mortgage rates. It also implies that expected appreciation is more complicated that trends in house price changes and mortgage rates, as it reflects expected inflation.

There were several limitations. The Shi was not designed for the United States (or California), and not all of the data was available. Also, many of the data series used were not
completely defined in the paper, leaving them open to interpretation by the reader. The Harris models were created before any bubble, however EX3 was a simple version of EX5.

It is too early to tell if there is indeed a positive relationship between interest rates in California or not, particularly as we can only speculate about the moving probability or other factors that might cause such behavior. However, there is evidence that housing in California features behavior that is complicated by more than just the bubble. Modeling expected appreciation will likely need to include both inflation and change in house prices, as well as more information regarding rentals (in particular, house rentals). We recommend that those who wish to understand housing in California should make a specific model for the state, rather than try to create something general.
VIII: References


### IX: Appendix

<table>
<thead>
<tr>
<th>Name</th>
<th>Used in Harris Model</th>
<th>Used in Shi Model</th>
</tr>
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<tbody>
<tr>
<td>10-Year Treasury Bill Yield, TB10 (%)</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>90-Day Treasury Bill Yield, TB90 (%)</td>
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<td>No</td>
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<td>Consumer Confidence Index, CCI (%)</td>
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<td>Yes</td>
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<td>Consumer Price Index, CPI</td>
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</tr>
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<tr>
<td>Vacancy (%)</td>
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Table 1: CCI, CPI, TB10, and TB90 are national. All other data is for California.

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<tr>
<th></th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Δ - Minimum</th>
<th>Δ - Maximum</th>
<th>Δ - Mean</th>
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Table 2: Descriptive Statistics of Data Used in Eq. 1
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<th>EX2</th>
<th>EX3</th>
<th>EX4</th>
<th>EX5</th>
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<td>-0.75</td>
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<td>(-2.72)*</td>
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<tr>
<td></td>
<td>(4.07)**</td>
<td>(3.69)**</td>
<td>(4.29)**</td>
<td>(5.26)**</td>
<td>(4.42)**</td>
</tr>
<tr>
<td>Vacancy</td>
<td>49527 (1.99)</td>
<td>23242 (1.2)</td>
<td>25782 (1.26)</td>
<td>27625 (1.38)</td>
<td>26300 (0.92)</td>
</tr>
<tr>
<td></td>
<td>(1.99)</td>
<td>(1.2)</td>
<td>(1.26)</td>
<td>(1.38)</td>
<td>(0.92)</td>
</tr>
<tr>
<td>Vacancy(1)</td>
<td>-220574 (-1.81)</td>
<td>-316180 (-3.05)**</td>
<td>-318110 (-2.65)**</td>
<td>-471248 (-3.67)**</td>
<td>-296785 (-2.40)*</td>
</tr>
<tr>
<td></td>
<td>(-1.81)</td>
<td>(-3.05)**</td>
<td>(-2.65)**</td>
<td>(-3.67)**</td>
<td>(-2.40)*</td>
</tr>
<tr>
<td>Mortgage</td>
<td>-42950 (-1.64)</td>
<td>16171 (0.85)</td>
<td>40720 (1.99)</td>
<td>-77673 (-2.35)*</td>
<td>56244 (1.97)</td>
</tr>
<tr>
<td></td>
<td>(-1.64)</td>
<td>(0.85)</td>
<td>(1.99)</td>
<td>(-2.35)*</td>
<td>(1.97)</td>
</tr>
<tr>
<td>Eq2</td>
<td>-27797 (-3.13)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.13)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eq3</td>
<td>1.2036 (4.53)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.53)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eq4</td>
<td>3713 (3.14)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.14)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eq5</td>
<td>112044 (3.35)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.35)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eq7</td>
<td>0.9422 (3.05)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.05)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.8007</td>
<td>0.8555</td>
<td>0.8012</td>
<td>0.8105</td>
<td>0.8017</td>
</tr>
</tbody>
</table>

*Table 3: Parameter Estimates with t-values (* - p-values < .05, ** - p-values < .01)*
The following code was used for the Shi model.

```sas
PROC IMPORT out=work.shi DATAFILE="shi.xlsx"
   DBMS=xlsx replace;
   SHEET="sheet1";
   GETNAMES=YES;
RUN;

data work.shi2;
   set work.shi;
   diff_log_real_rent_lag = lag(diff_log_real_rent);
   diff_rir_lag = lag(diff_rir);
   delta_unemployment_lag = lag(delta_unemployment);
   diff_cci_lag = lag(diff_cci);
RUN;

proc syslin data=work.shi2 out=pred;
   model diff_log_real_hp = lag_hp diff_log_real_rent
      diff_log_real_rent_lag diff_rir diff_rir_lag delta_unemployment
      delta_unemployment_lag diff_cci diff_cci_lag;
   output p=yhat;
RUN;

proc means data=work.pred;
   var diff_log_real_hp diff_log_real_rent diff_rir
      delta_unemployment diff_cci;
RUN;

proc autoreg data=work.shi2;
   model diff_log_real_hp = / stationarity=(adf);
RUN;

title "Shi Model of House Price";
title2 "Log Real House Price with Log Real Rent";
axis1 label=("Year") order=(1991 to 2015 by 4) minor=none offset=(1,1);
axis2 label=(angle=90 "Log Real House Price (Diff)") order=(-.21 to .1
   by 0.031) minor=none;
axis3 label=(angle=90 "Log Real Rent (Diff)") order=(-.05 to .026 by
   0.0076) minor=none;
legend1 label=none value=(color=blue height=1 'Difference of Log Real
   House Price' 'Predicted Value') frame;
legend2 label=none value=(color=blue height=1 'Difference of Log Real
   Rent' 'Lag of Difference of Log Rent') frame;

proc gplot data=pred;
   plot diff_log_real_hp*year=1
      yhat*year=2 / overlay legend=legend1 haxis=axis1
   vaxis=axis2;
   plot2 diff_log_real_rent*year=3
      diff_log_real_rent_lag*year=4 / overlay legend=legend2
   vaxis=axis3;
RUN;
```
**SAS Code:** The following code was used for the Harris model and for most plots.

```sas
proc import out=work.datatemp DATAFILE="harris.xlsx"
   DBMS=xlsx REPLACE;
   SHEET="Sheet1";
   GETNAMES=YES;
run;

data hp;
   set datatemp;
   house_price_nom_diff = dif(median_house_price);
   appreciation = dif(median_house_price) / lag(median_house_price) * 100;
   house_price_nom_log = log(median_house_price);
   house_price_nom_diff_log = dif(log(median_house_price));
   cpi_diff = dif(cpi);
   vacancy_lag = lag(vacancy);
run;

title "Autoregression of House Prices";
title2;
proc autoreg data=hp;
   model median_house_price = year;
run;

/*Calculate ARIMA(1,0,1) forecasts for house prices*/
title "Forecast of Median House Price ARIMA(1,0,1)"
proc arima data=hp;
   identify var=median_house_price;
   estimate q=1 p=1;
   forecast lead=1 out=hpfc id=year;
run;
quit;

data housing_bubble;
   merge hp (in=hpin)
       hpfc (in=hpfcin);
   by year;
   if hpin and hpfcin;
   house_diff = forecast - median_house_price;
run;
quit;

data ex1;
   proc means kurtosis data=housing_bubble;
   var house_diff;
run;

/*Expectations 1: ARMA of past inflation*/
title "EX1: Past Inflation"
proc arima data=hp;
   identify var=cpi(1);
   estimate q=0 p=2;
   forecast lead=1 out=armacpi id=year;
run;
quit;
data ex1;
```
merge armacpi (in=arin)
   hp(in=cain);
by year;
if arin and cain;
run;
title "House Price (EX1)"
proc pdlreg data=ex1;
   model median_house_price = median_household_income vacancy(1)
mortgage_contract_rate forecast;
   output out = hp_ex1 p=hp_yhat ucl=upper lcl=lower;
run;
proc means data=hp_ex1;
   var median_house_price median_household_income vacancy
mortgage_contract_rate forecast hp_yhat;
run;
/*Expectations 2: past price appreciation*/
title "EX2: Past Price Appreciation"
proc arima data=hp;
   identify var=house_price_nom_diff(1);
   estimate q=0 p=4;
   forecast lead=1 out=armappa id=year;
run;
quit;
data ex2;
   merge armappa (in=arin)
   hp(in=cain);
   by year;
   if arin and cain;
run;
title "House Price (EX2)"
proc pdlreg data=ex2;
   model median_house_price = median_household_income vacancy(1)
mortgage_contract_rate forecast;
   output out = hp_ex2 p=hp_yhat ucl=upper lcl=lower;
run;
proc means data=hp_ex2;
   var median_house_price median_household_income vacancy
mortgage_contract_rate forecast hp_yhat;
run;
/*Expectations 3: Distributed Lags of Past Appreciation*/
/*Harris models past appreciation with house price increases. This is
EX3. */
title "EX3: Distributed Lags of Past Appreciation"
proc pdlreg data=hp;
   model appreciation = house_price_nom_diff;
   output out = ex3 p=forecast ucl=upper lcl=lower;
run;
title "House Price (EX3)"
proc pdlreg data=ex3;
model median_house_price = median_household_income vacancy(1) mortgage_contract_rate forecast;
output out = _hp_ex4 p=_hp_yhat ucl=upper lcl=lower;
run;
proc means data=_hp_ex3;
var median_house_price median_household_income vacancy mortgage_contract_rate forecast _hp_yhat;
run;
/*Expectations 4: Distributed Lags of Past Inflation*/
title "EX4: Distributed Lags of Past Inflation"; proc model data=_hp_list;
parms int a1 a2 a3;
%pdl(cpi_diff,3,3);
tb10 = int + a1*tb3 + a2*cpi_diff + %pdl(cpi_diff,year);
id year;
fit tb10 / out=model1 outpredict converge=1e-6 prl=both;
run;
quit;
data ex4model (keep=year forecast);
set model1 (rename=(TB10=forecast));
run;
data ex4;
merge hp (in=cain) ex4model (in=e4in);
by year;
if cain and e4in;
run;
title "EX4: Distributed Lags of Past Inflation";
proc sgplot data=ex4;
yaxis grid label="Expected Appreciation Based on Past Inflation";
scatter x=year y=forecast/ markerattrs=(symbol=circle size=6) legendlabel="Expected Appreciation" name="yhat";
series x=year y=tb10 / lineattrs=GraphFit legendlabel="Inflation" name="actual";
keylegend "yhat" "actual" / across=4 noborder position=TopRight location=inside;
run;
title "House Price (EX4)";
proc pdlreg data=ex4;
model median_house_price = median_household_income vacancy(1) mortgage_contract_rate forecast / all;
output out = _hp_ex4 p=_hp_yhat ucl=upper lcl=lower;
run;
proc means data=_hp_ex4;
var median_house_price median_household_income vacancy mortgage_contract_rate forecast _hp_yhat;
run;
title "Harris Model for House Price";
proc sgplot data=_hp_ex4;
yaxis grid label="House Price ($)";
xaxis grid label="Year" values=(1990 to 2016 by 2);
band x=year upper=upper lower=lower /
transparency=0.5 legendlabel="95% Confidence" name="conf";
scatter x=year y=hp_yhat / markerattrs=(symbol=circle size=6)
legendlabel="Expected House Price" name="yhat";
series x=year y=median_house_price / lineattrs=GraphFit
legendlabel="House Price" name="actual";
keylegend "conf" "yhat" "actual" / across=4 noborder
location=inside;
run;

/*Compare Harris and Shi models graphically*/
proc import out=work.shi_end DATAFILE=shi_final_values.xlsx"
   DBMS=xlsx REPLACE;
   SHEET="Sheet1";
   GETNAMES=YES;
run;

data shi_harris;
   merge shi_end (in=shiin) hp_ex4 (in=hpin);
   by year;
   if shiin and hpin;
run;

proc means data=shi_harris;
   var median_house_price;
run;

title "Comparison of Shi and Harris Models";
axis1 label=($(Year$) order=(1992 to 2015 by 2) minor=none offset=(1,1));
axis2 label=($(angle=90 "House Price ($)" order=(170000 to 570000 by 40000)) minor=none;
legend1 label=none value=(color=blue height=1 'House Price' 'Predicted Value (Harris)' 'Predicted Value (Shi)') frame;

proc gplot data=shi_harris;
   plot median_house_price*year=1
        hp_yhat*year=2
        shi_house_price*year/ overlay legend=legend1
haxis=axis1 vaxis=axis2;
run;